

renew it till by the recovery of the Suns light they had recovered their former gayety and mirth : However we cannot learn that any Star besides that of *Venus* was discovered by those which were spectators of it in the open air.

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### III. *The Dimension of the Solids generated by the Conversion of Hippocrates's Lunula, and of its Parts about several Axes, with the Surfaces generated by that Conversion, by Ab. De Moivre, F. R. S.*

**L** Et B C A (Fig. 1.) be an Iſoſcelles Triangle right angled at C. with the Center C, and diſtance CB, deſcribe the Quadrant B F A; on B A, as a Diameter, deſcribe a Semicircle B K A; the Space comprehended between the Quadrantal arc B F A, and the Semicircumference B K A, is call'd *Hippocrates's Lunula*.

If upon B C you take my two Points D, E, and draw the Perpendiculars D H, E M, meeting B A in I & L, and cutting a Portion F G M H of the *Lunula*; the Solid generated by the conversion of this Portion about the Axis B C, is equal to a Priſm where Baſe is I L M H, and height the Circumference of a Circle whoſe Diameter is B C; and the Solid generated by the Semicircle B K A, is equal to a Priſm or Semicylinder, whoſe baſe is the Semicircle B K A, and height the Circumference of a Circle whoſe Diameter is B C.

Having biſected B A in R, and B C in P, the Surface generated by the conversion of the Arc H M about the Axis B C, is equal to  $\pi \times B P \times H M + B R \times D E$  (ſuppoſing the ratio of the radius to the Circumference to

to be as  $r$  to  $c$ ) and the Surface generated by the Semicircumference  $BKA$  is equal to a Rectangle whose base is the sum of that Semicircumference and Diameter  $BA$ , and height the Circumference of a Circle whose Diameter is  $BC$ . As for the Surface generated by the arc  $GF$ , 'tis well known, that it is equal to a Rectangle whose base is the Circumference of a Circle whose Radius is  $BC$ , and height  $DE$ ; Therefore the Surface generated by the Conversion of the Portion  $M H F G$  is known.

If upon  $BA$  (Fig. 2.) you take any two Points  $I, L$ , and draw  $IN, LV$  perpendicular to it, cutting the Quadrant in  $O$  and  $T$ , and the Circumference in  $N$  and  $V$ , the Solid generated by the conversion of the Portion  $ONU T$  about the Axis  $BA$ , is equal to a Prism whose Base is  $IOTL$ , and height the Circumference of a Circle whose Diameter is  $BA$ .

Having bisected  $BA$  in  $R$ , and drawn  $CR$  meeting the Quadrant in  $G$ , the Surface generated by the Conversion of the Arc  $OT$  about  $BA$  is equal to  $\frac{1}{2} \times CG \times IL - CR \times OT$ .

Bisect  $DE$  in  $Y$  (Fig. 1.) through the Center  $R$  draw  $SQ$  parallel to  $BC$ , meeting the Circumference  $BKA$  in  $S$ ,  $BK$  parallel to  $AC$  in  $V$ , and the Lines  $DH, EM$  in  $N$  and  $O$ ; the Solid generated by the Conversion of the Portion  $FGMH$  about the Axis  $AC$ , is  $\frac{1}{2} \times MO^3 - \frac{1}{3} NH^3 + PC \times NOMH + CY \times DNOE - \frac{1}{3} EG^3 + \frac{1}{3} DF^3$ , and the Solid generated by the Segment  $KBS$  is  $\frac{1}{2} \times VK^3 + PC \times BUKS$ . Therefore the Solid generated by the Semicircle  $BKA$  about  $AC$  is  $\frac{1}{2} \times PC \times VQAK + \frac{1}{2} \times BCQV - \frac{1}{3} AC^3 + \frac{2}{3} VK^3 + PC \times BVKS$ , which by due reduction will be found equal to the Solid generated by the Conversion of the same Semicircle about the Axis  $BC$ .

The Solid generated by the Portion  $ONVT$  about the Axis  $CD$ , is equal to  $\frac{c}{r} \times \frac{1}{3} LV^3 - \frac{1}{3} IN^3 - \frac{1}{3} QT^3 + \frac{1}{3} PO^3 + CS \times PQIL$ .

From the Points  $M, H$ , drop the two perpendiculars  $MZ, HW$ , upon  $CA$  prolong'd if need be; the Surface generated by the Conversion of the Arc  $HM$  about the Axis  $CA$  is equal to  $\frac{c}{r} \times PC \times HM - RA \times WZ$ , when the point  $Z$  is next to  $C$ , or  $\frac{c}{r} \times PC \times H + MRA \times WZ$  when the point  $W$  is next to it.

Those that will think it worth their while to bestow some little pains to find the Demonstration of this, may solve the following Problem.

Any two Conic Sections being given, forming a *Lunula* by their Interfection, and a right line being given by position, about which, as an Axis, this *Lunula* is imagined to turn, to find the Solids generated by the Conversion of any of its parts, cut off by lines perpendicular to that axis, or parallel to it, or making any given Angle with it, as also the Surfaces made by that Conversion.

fig: 3.





